דפי מבחן/בוחן

מקצוע _____ נושא _____

שם התלמיד/ה _____ כיתה _____ תאריך _____

ביה"ס התיכון הראל
מבשרת ציון

1

$x^2+y^2-6x+5=0$ — I

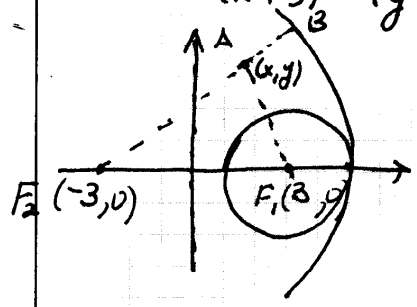
$x-2\cdot x\cdot 3+3^2-3^2+y^2+5=0$

$(x-3)^2+y^2=4$ ⇒ $(3,0)$ $R=2$

$x^2+y^2+6x-55=0$ — II

$y^2+x^2+2\cdot x\cdot 3+3^2-3^2-55=0$

$(x+3)^2+y^2=64$ $(-3,0)$ $R=8$



$F_2A+AF_1=8-x+2+x$

$F_2A+AF_1=10$

לכן מקום של המוקדים של האליפסה

$2a=10 \Rightarrow a=5$

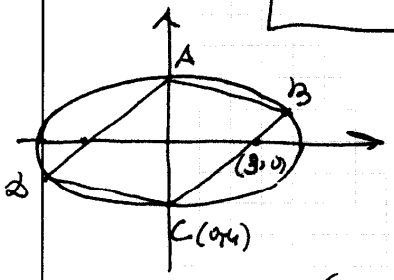
$c=3$

$a^2=b^2+c^2 \Rightarrow b^2=a^2-c^2$

$b=\sqrt{25-9}=\sqrt{16}=4$

$$\frac{x^2}{25}+\frac{y^2}{16}=1$$

2



$m=\frac{0-(-4)}{3-0}=\frac{4}{3}$

$l_{BC}: y=\frac{4}{3}(x-3)$

$y=\frac{4}{3}x-4$

$\frac{x^2}{25}+\frac{(4x-12)^2}{144}=1 \Rightarrow \frac{x^2}{25}+\frac{(x-3)^2}{9}=1$

$9x^2+25(x-3)^2=225$

$9x^2+25x^2-150x+225=225$

$34x^2-150x=0$

$x=0$

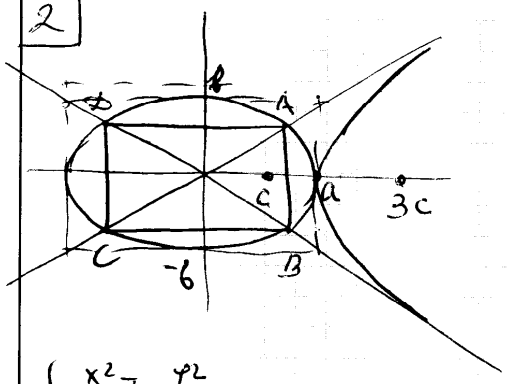
$x=\frac{75}{17}=4.412$

$S_{ABCD}=4.412\cdot 8=35.294$

מבשרת ציון

שם התלמיד/ה _____ כיתה _____ תאריך _____

2



$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$$

$a^2+b^2=(3c)^2$ — הפרבולה

$b^2+c^2=a^2$ — אליפסה

$y=\pm\frac{b}{a}x$.

$$\begin{cases}\frac{x^2}{a^2}+\frac{y^2}{b^2}=1\\ y=\frac{b}{a}x\end{cases}\Rightarrow$$

$4c^2=b^2$

$a^2=5c^2$

$$\frac{x^2}{a^2}+\frac{\left(\frac{b}{a}\right)^2x^2}{b^2}=1\Rightarrow\frac{x^2}{a^2}+\frac{x^2}{a^2}=1\Rightarrow 2x^2=a^2$$

$x=\frac{\sqrt{2}}{2}a\Rightarrow BC=\sqrt{2}a$ — 20%.

$y=\frac{b}{a}\cdot\frac{\sqrt{2}}{2}a=\frac{\sqrt{2}}{2}b\Rightarrow AB=\sqrt{2}b$ — 20%.

$$S_{ABCD}=\sqrt{2}a\cdot\sqrt{2}b=2ab=16\sqrt{5}$$

$$ab=8\sqrt{5}$$

$$\begin{cases}a^2+b^2=9c^2\\ a^2-b^2=c^2\end{cases}\quad-\begin{cases}a^2+b^2=9c^2\\ 9a^2-9b^2=9c^2\end{cases}$$

$8a^2-10b^2=0\qquad 4a^2-5b^2=0.$

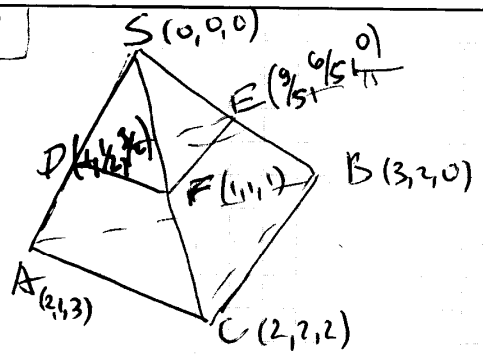
$$\begin{cases}2a=\sqrt{5}b\Rightarrow a=\frac{\sqrt{5}}{2}b\\ ab=8\sqrt{5}\end{cases}\Rightarrow\frac{\sqrt{5}}{2}b\cdot b=8\sqrt{5}$$

$b^2=16\Rightarrow\boxed{b=4}\Rightarrow\boxed{a=2\sqrt{5}}$

$c^2=a^2-b^2\Rightarrow c^2=20-16=4\quad\boxed{c=2}$

$$\boxed{\frac{x^2}{20}+\frac{y^2}{16}=1}\qquad\boxed{\frac{x^2}{20}-\frac{y^2}{16}=1}$$

3



$\overline{SF} = \underline{n} = (1,1,1)$

$\pi : [(x,y,t) - (1,1,1)] \cdot (1,1,1) = 0$

$\boxed{x+y+z-3=0}$

$\ell_{SB} : \underline{x} = t(3,2,0)$ $\quad E : 3t+2t-3=0 \Rightarrow t = \frac{3}{5}$

$E\left(\frac{9}{5}, \frac{6}{5}, 0\right)$

$\ell_{SA} : \underline{x} = t(2,1,3)$

$D : 2t+t+3t-3=0 \Rightarrow t = \frac{1}{2}$ $\quad D\left(1, \frac{1}{2}, \frac{3}{2}\right)$

$h = \left|\frac{3}{\sqrt{3}}\right| = \sqrt{3}$

$\overline{FE} = \left(\frac{9}{5}-1, \frac{6}{5}-1, -1\right) = \left(\frac{4}{5}, \frac{1}{5}, -1\right)$

$\overline{FD} = \left(0, -\frac{1}{2}, \frac{1}{2}\right)$

$$S_{\triangle DEF} = \frac{1}{2}\left\| \begin{vmatrix} i & j & k \\ \frac{4}{5} & \frac{1}{5} & -1 \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{vmatrix} \right\| = \frac{1}{2}\left| \frac{1}{10}i - \frac{2}{5}k - \frac{1}{2}i - \frac{2}{5}j \right| =$$

$$\begin{vmatrix} i & j & k \\ \frac{4}{5} & \frac{1}{5} & -1 \end{vmatrix}$$

$$= \frac{1}{2}\left| -\frac{2}{5}i - \frac{2}{5}j - \frac{2}{5}k \right| =$$

$$= \frac{1}{2}\sqrt{3 \cdot \frac{4}{25}} = \frac{1}{5} \cdot \sqrt{3} \qquad S_6 = \frac{1}{5} \cdot \sqrt{3}$$

$$V = \frac{1}{3} \cdot \sqrt{3} \cdot \frac{1}{5} \cdot \sqrt{3} = \frac{1}{5}$$

4) $i\bar{z}\cdot z = z - \bar{z}$ $\quad |z|=1$ $\quad$ k

$i = z - \bar{z}$ $\quad |z|^2 = z\cdot\bar{z} \Rightarrow$

$i = x+yi - x+yi$ $\quad z\cdot\bar{z}=1$

$2y = 1 \Rightarrow y = \frac{1}{2} \Rightarrow x^2+y^2=1 \Rightarrow$

$x_1 = \frac{\sqrt{3}}{2}$, $x_2 = -\frac{\sqrt{3}}{2}$.

$z_1 = \frac{\sqrt{3}}{2} + \frac{1}{2}i$ $\quad z_2 = -\frac{\sqrt{3}}{2} + \frac{1}{2}i$

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n

$z^n = \bar{z}$ $\quad n \neq 1$, $z \neq 0$.

$|z| = |\bar{z}| \Rightarrow |z^n| = |\bar{z}|$

$|z^n| = |z|^n \Rightarrow |z|^n = |z| \;/:|z|$

$|z|^{n-1} = 1 \Rightarrow |z| = 1$

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c

$z^4 = \bar{z}$

$|z|^4 = |\bar{z}| \Rightarrow$ $\quad \Leftarrow$

$|z| = 1 \Rightarrow (\mathrm{cis}\,\alpha)^4 = \mathrm{cis}(-\alpha)$

$(\mathrm{cis}\,\alpha)^5 = 1$

$5\alpha = 360k \Rightarrow \alpha = 72k$

$z_k = \mathrm{cis}\,72k$

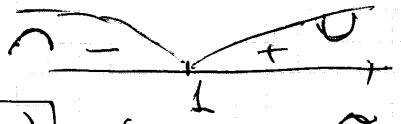
$k = 0, \ldots, 4$

5] $a>1$ $\quad f(x)=a(x+1)e^{-x}+x$ (c)

$f'(x)=a\left(e^{-x}-e^{-x}(x+1)\right)-1=a\left(-xe^{-x}\right)-1$

$f''(x)=-a\left(e^{-x}-e^{-x}\cdot x\right)=ae^{-x}(x-1)$

$f''(x)=0 \Rightarrow x=1$



$\leftarrow$ (2) −

$\left(1,2ae^{-1}-1\right)$ − ∫c : [illegible]

(d)

$f(a)=6e^{-a}-a$

$f(a)=a(a+1)e^{-a}-a=6e^{-a}-a$

$a(a+1)=6$

$a^2+a-6=0 \Rightarrow \boxed{\begin{array}{l} a_1=2 \\ a_2=-3 \end{array}} \leftarrow$ ✗

$a>1$

$V=\pi\int_t^{2t} x\cdot e^{-x^2}dx=-\frac{1}{2}\pi\int_t^{2t} e^{-x^2}d(-x^2)=$ (3) $t>0$

$=-\frac{1}{2}\pi\ e^{-x^2}\Big|_t^{2t}=-\frac{1}{2}\pi\left(e^{-4t^2}-e^{-t^2}\right)$

$V'(t)=-\frac{1}{2}\pi\left(e^{-4t^2}\cdot(-8t)-e^{-t^2}(-2t)\right)=$

$=+\frac{1}{2}\pi t\cdot 2\left(4e^{-4t^2}-e^{-t^2}\right)=$

$=\pi t\cdot e^{-t^2}\left(4e^{-3t^2}-1\right)$

$-3t^2=\ln\frac{1}{4} \quad t^2=\frac{2\ln 2}{3} \Rightarrow$

$4t^3=1$

$t^3=\frac{1}{4}$

$t=\sqrt[3]{\frac{1}{4}}=e^{-x^2}$

$e^{-3t^2}=\frac{1}{4}$ $\quad -x^2=\frac{1}{3}\ln 2$

$$t=\sqrt{\frac{\ln 4}{3}}\approx 0.67977$$