

Why single-ended tube amplifiers?

About distortion behavior between SE amplifiers and speakers

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1. Introduction

Single-ended triode tube amplifiers are not that hard to find anymore. You can buy them from many companies, or you can build one with parts that are offered by dozens of suppliers. The high-end audio press has been talking about them for the past several years and it is very strange that something that goes against all the traditional ways of designing, building and evaluating amplifiers has stayed with us for such a long time. We should be shocked by how good a single ended tube amplifier can sound when paired with a compatible loudspeaker. It can sound like music and at the same time it can exhibit very high distortion and very low output power on the test bench.

To say that people enjoy distorted or heavily colored sound does not seem to be a reasonable answer. If we take a look at who has been using these amplifiers, we will have to agree that among them we can find many highly respected listeners who have a keen ear and years of live music exposure. There must be some very good reasons for this apparent paradox. What I intend to do in this article is to suggest and explain one important reason which I believe has not been given enough attention up till now. After about five years of designing and building amplifiers with single-ended circuits and the loudspeakers to work with them as a system, I would like to describe some of the things I have observed. They may be useful in solving the giant puzzle that tubed single-ended amplifiers have presented to us.

2. Some Good Reasons

Looking back at what has been said about this subject -- and leaving the "people like second harmonic distortion and colored sound" type of argument aside -- we are left with many important articles, which touch on the subject of distortion in amplifiers in general, and some about the specific single-ended case. The articles by Norman Crowhurst reprinted in *Glass Audio* (The Amplifier Distortion Story part I & II, GA 6/95 & 1/96) and the article by Lynn Olson, in *Valve* (12/97) have several reasons for the good sound of amplifiers without feedback. They are very important for understanding the true meaning of the distortion specifications in power amplifiers. The articles by Scott Frankland in

Stereophile (SE vs. PP - starting 12/96) are very good references about all the problems and benefits of single-ended topology. This may have its roots in the old question of tube versus transistor sound that is very well covered by the 1972 AES paper written by Russell Hamm. It points to many important effects and explains the difference between tube and transistor amplifiers. The paper looks at the overload characteristics of tubes, transistors and integrated circuits and describes several issues related to this behavior, including some psychoacoustic ones. It helps to explain why tubes may sound better than transistors. This same paper explains why tube amplifiers usually can play louder than a transistor amplifier of the same specified output power before we reach for the volume control to turn it down.

Although the arguments used by Mr. Hamm can also be used to partially clarify the non-feedback single-ended tube amplifier case, a lot is left to be explained. Mr. Hamm was looking at the reasons why tube amplifiers sounded different (and usually better) than transistor amplifiers. But these tube amplifiers were, as a group, very well engineered and had very decent specifications. They had been designed following the rules of the low distortion/low output impedance paradigm and the main difference in sound between them and transistor amps could be related to the way they behave when overloaded. Looking backwards, we may think that part of the difference in sound could also be explained by the fact that tube circuits change their operating point with age - especially with the passive components of the 50's and 60's - and the chances that a tube amplifier would run at some point in its life with some imbalance were certainly high. But in any controlled test, the first thing you should do is to make sure that your equipment is within the specifications! Therefore if the imbalance was one of the reasons why a tube amplifier sounded better, people would never find it in controlled tests. On the other side, most early single-ended amplifiers have been built entirely by ear, in complete disregard to the low distortion/low output impedance paradigm. The better sounding ones always seemed to have high distortion when finished. More than once have we heard single-ended amplifier designers comment that "whenever some of the specifications got better, the sound got worse" .

Almost all of the dozens of articles that I have read in favor of single-ended amplifiers try to show that even with the high distortion and high output impedance, there are a lot of good reasons for them to sound good. Yes, they should sound good **despite** these awful specifications. And there are some very good points in these comments. I have tried to list all apparently correct technical reasons for the good sound of single-ended amplifiers given in several articles and to broadly classify them. After doing it, I finally arrived at five great groups:

- SE amps usually have extremely simple circuits with a very low parts count. With all the current vision that passive components do affect the sound, it is easy to conclude that the less the better.
- Most single ended amps do not use global negative feedback. Negative feedback can improve the measured distortion levels but usually change considerably the distortion spectrum and may produce serious problems with complex loads and during recovery from overload.
- There is no need for a phase splitter stage as in push-pull tube designs. The signal does not need to be inverted and recombined at the output stage. It is not hard to agree that it should be better to avoid processing the audio signal this way.
- A single ended output stage has to work in pure class A always. There is no way to make a single ended output stage work in any other class (AB or B). These other classes of operation require, by definition, that we switch the conduction from one

- device to another and in a single ended output stage there isn't another device.
- The output transformer will be very different. It will not need to rely on the balance between each half of the primary and will have an air gap that changes favorably its distortion spectrum.

3. and One More Reason

Here I will take the opposite approach and show that not despite but **due to** the high distortion (and also due to the high output impedance), single-ended amplifiers can be a sensible way of putting together a system with a performance that in some areas, with present day technology, may be extremely expensive or out of reach using the traditional methods of designing amplifiers and loudspeakers.

What follows from here needs one comment. I am assuming that people always prefer the sound from a system that has lower distortion. Lower distortion of any kind! I can see no reason for this not be true. I agree that psychoacoustic tests may find that some kind of distortion is good or may improve a particular aspect of the reproduction, but I believe that in the long run, we all look for *less* distortion. But, in any case, even if we are looking for some distortion effect, everything that I will say will still hold true and may even help us to achieve this target.

4. Total System Distortion

What is the distortion that we actually hear? I think everyone will agree that we hear the distortion that reaches our ears. There are several types of distortion. Harmonic distortion, intermodulation distortion, frequency distortion, phase distortion, time distortion, dispersion distortion and so on. And they are usually interrelated. It is easy to see that we can quickly get lost with all this. But we have agreed on something very important. *It is the distortion that gets to our ears that matters.* We will see why I am repeating this in a while. This distortion is the addition of all the distortions in the sound system. We should add the distortion from the microphone, mic preamp, mixer, recording gear, mastering gear, the medium (tape, CD, LP or whatever), the source (tape, CD, turntable/arm/cartridge or whatever), preamp, amplifier and finally the loudspeaker and the room.

When dealing with amplifiers, the total harmonic distortion (THD) has always been the preferred specification for them. And the high measured THD is the main criticism that single-ended amplifiers have to face. Therefore, let's examine its case.

What is the best way to obtain the lowest distortion sound at our ears? Make everything perfect! No distortion at all. This is much more easily said than done! Although most parts of a state of the art digital based system can be made to have very low measured harmonic distortion, this cannot be said of the speakers. At the present technology, moving coil drivers will have much higher THD than any other electronic device in a system. And if we do not count the room, the loudspeaker is the last element in the system. The total harmonic distortion that reaches our ears is the sum of the whole system distortion up to the amplifier plus the loudspeaker distortion. And in our state of the art digital based system with a very low distortion amplifier most of the *total system* THD we will measure will be the loudspeaker distortion. But still we can hear the sound of different low distortion amplifiers, DACs or any other low distortion piece of equipment.

Why can this happen ? Moving coil speakers drivers are essentially a very simple electro-mechanical system and they normally have a very high total harmonic distortion

specification compared to any other device in this digital based chain. But **if we examine the distortion spectrum of any reasonable speaker we will see that all the distortion is low order. In most cases it is mainly second harmonic with some third harmonic also appearing with lower levels. The higher order terms will only become noticeable near the limit of power.** On the other side the so called low distortion electronics may have high order harmonic distortion (and sometimes transient and/or non harmonic related distortions) that even in relatively small amounts may be noticed by the ear. This will not be shown as a high THD reading. And they will come through the speaker unmasked because of the low order distortion spectrum that most loudspeakers present.

What will happen when we put a high distortion tube single ended amplifier in the same system in place of the low distortion amplifier ? Single ended tube amplifiers are very simple electronic devices. They normally have a high harmonic distortion specification. But **if we examine the distortion spectrum of most single ended amplifiers we will see that all the distortion is low order. In most cases it is mainly second harmonic with some third harmonic also appearing with lower levels. The higher order terms will only become noticeable near the limit of power.** I have repeated the same words as above when describing the speaker driver distortion because this is the key. Single ended amplifiers usually do not have problems like high order or transient distortions and have low order distortions just like loudspeakers. If we understand that second harmonic distortion is a mathematical construction that happens to describe an aspect of nature and has a precise definition that does not take in account if it was generated electrically or mechanically or by whatever means, we can take a look at how do the distortion from the single ended amplifiers adds to the loudspeaker distortion and I believe we will have discovered where many pieces of our puzzle should be placed.

Even if you do not like mathematics you should look at the Appendix where I show how second harmonic distortion will sum. This is not at all intuitive at first sight and I will resume in plain words some of what all the math tells:

If two devices have only second harmonic distortion in the same quantity (lets say 1%) what happens if we connect them one after the other ?

The result will depend on the phase of this second harmonic in relation to the fundamental in each of the devices. (this phase can be anything from zero to 360° or if you prefer from -180° to $+180^\circ$, always spanning a 360° range). At this point I should say that the multiplication will generate 3rd and 4th harmonics but at much lower levels. Let's look at the 2nd harmonic:

- If the two devices are just equal, with the same phase relation between the distortion and the fundamental, the result will be a total of 2.0% of 2nd harmonic distortion.
- A 30° difference in the relation of the 2nd harmonic and the fundamental between the two devices will produce a total of 1.93% of 2nd harmonic distortion.
- A 60° difference will produce a total of 1.73% of 2nd harmonic distortion.
- A 90° difference will produce a total of 1.41% of 2nd harmonic distortion.
- A 120° difference will produce a total of 1.0% (exactly the same distortion as any of the devices alone)
- A 150° difference will produce a total around 0.52% (about half of the distortion of any of the devices alone)
- If there is 180° difference in the relation of the 2nd harmonic and the fundamental between the two devices the sum will be zero 2nd harmonic (yes that is right, in this

ideal situation there would be NO 2nd HARMONIC DISTORTION). (*)

(*) The total distortion cancellation of the second harmonic may not occur if the complete output of the amplifier with 2nd harmonic allows DC to pass and the distortion producing mechanisms are DC coupled to the output. Then some 2nd harmonic will result that will not be cancelled. But it should only be noticeable at very high levels since it has a coefficient with E^4 .

But what are the chances that we can get this complete distortion cancellation, as it is normally called ? Very few, just like there are very few chances that the distortions will fully add so we would have 2.0%. If the difference between the phases of the 2nd harmonics is between 0° and $\pm 120^\circ$ there will be an increase in the total system distortion compared to the distortion of one device alone up to a maximum of 2.0%. If the difference is between $\pm 120^\circ$ and $\pm 180^\circ$ there will be a reduction in the total system distortion down to a minimum of zero! Of course the audibility of different levels of distortion may not follow the same relation as these numbers suggest, but there is a great possibility that the reduction may be more noticeable than the increase.

And what are the chances that we have one condition or the other ? Let's start by imagining that for any frequency the phase of the second harmonic could be any, with no preferred behavior. We could say that we would have a 66% ($2 \times 120 / 360 = 2/3$) chance of increasing the distortion over one device distortion alone and a 33% ($2 \times (180 - 120) / 360 = 1/3$) chance of decreasing. Real world single ended amplifiers and real world speakers will definitely show one preference for one case or the other at several frequency ranges. And now we have the really great thing about all this: *If there is more distortion in the system than in the speaker alone in most of the frequency range, just reverse the polarity of the connection between the amplifier and the speaker (This is the ONLY place to invert it) and you will have shifted 180° the amplifier. Now you have a good chance of getting less distortion than you started with and probably less distortion than the speaker alone! This is usually clearly audible.* (You can flick the polarity switch in your DAC to restore absolute polarity. This has nothing to do with it. It also has nothing to do with reversing the primary windings of the output transformers. It may change the relation of several important parameters to ground altering the frequency response). We can say that including the possibility of reversing the polarity, at any given frequency, we would have a 66% chance of decreasing the distortion compared to the distortion of one device alone against a 33% chance of increasing it.

Obviously things could not be that simple. I have described an ideal case. There are three things to remember. First I am talking about one frequency. When you change the polarity you changed it for all frequencies. Second, what I have just said assumes that the two devices (amplifier and loudspeaker) have the same amount of 2nd harmonic distortion. Third, I have assumed that the first device (amplifier) distortion is independent of the reversal of the load presented by the second device (loudspeaker). In real world conditions these two last facts are not completely true, but nevertheless all I have described occurs partially at all frequencies all the time you hook a single ended amplifier with a loudspeaker and sometimes it produces music!

Several techniques for increasing the distortion reduction, extending it to most frequency ranges, and improving the power tracking, can be used, including some very sophisticated ones. I believe that most of the tube swapping and several transformer coupling techniques own a good part of their success to this decrease in the total system distortion. But the easiest thing you can do is to invert your connection between the output stage and

the loudspeaker (This is the only place to invert the connection if you are looking for this effect). If you try to invert the connection of your single ended amplifier with your speaker you certainly will hear a difference. Sometimes it could be hard to tell which is best because you may lower the distortion at one frequency range and increase at another. Usually there is definitely a preferred way of hooking the amplifier. But you can be sure of one thing, this effect I described shows that by looking at what is often referred as its weakness (the high harmonic distortion) a single ended amplifier can be better in a system than the "straight (perfect) wire with gain" utopia or than any perfect amplifier! The fact that a single ended amplifier produces 2nd harmonic distortion in quantities close to a loudspeaker is the fundamental reason why the system can have less total 2nd harmonic distortion if the relative phases happen to be on the "good" side. And you can always try to find the preferred polarity of the connection. It is not uncommon that, although differently, both connections may sound better than a normal low distortion amplifier because the reductions will occur at different frequency ranges but may be more noticeable to the ear than the increase in distortion. Also do not forget that these amplifiers are usually free of other defects like high order harmonics, transient distortions and instabilities

The reduction of the other harmonics (from 3rd up) may also happen, but the conditions and requirements certainly are not as simple as the ones for 2nd harmonic. It seems to be much more difficult to happen by chance and to be level dependent. This deserves further investigation.

All the math in the appendix is for 2nd harmonic distortion and is valid for one frequency. The way the distortion varies with the frequency and the way it "breaths" with the power levels will be very important to how effective the effect can be. But all this certainly explains why single ended amplifiers are one of the better ways to have a low distortion system.

In figures 1, 2 and 3 we can see what happens with a loudspeaker driven by a low distortion amplifier and by a single ended amplifier of high 2nd harmonic distortion with both polarities. In this case the speaker alone has around 2% of distortion between 100 Hz and 1000 Hz with 2.83 Vrms in the input (that corresponds to 1W in an 8 ohm load). We can see this in figure 1. In figure 2 we connect it to a SE amplifier with an average 0.8% 2nd harmonic distortion at this output level. It is easy to see the distortion addition between 150 Hz and 300 Hz and subtraction around 900Hz. We could say that, as an average, the distortion has increased. Figure 3 shows the same set up with the other polarity. We can see that the whole 150 Hz till 300 Hz region has reduced distortion and that around 900 Hz we have an increase in it. The overall effect seems to be a reasonable decrease in distortion. I have chosen this example because it is not one of a carefully optimized system but one that shows that the effect may happen differently at different frequency bands. I have shown only the 2nd harmonic plots but the THD plots are just about the same. At this power level distortion is mainly 2nd harmonic

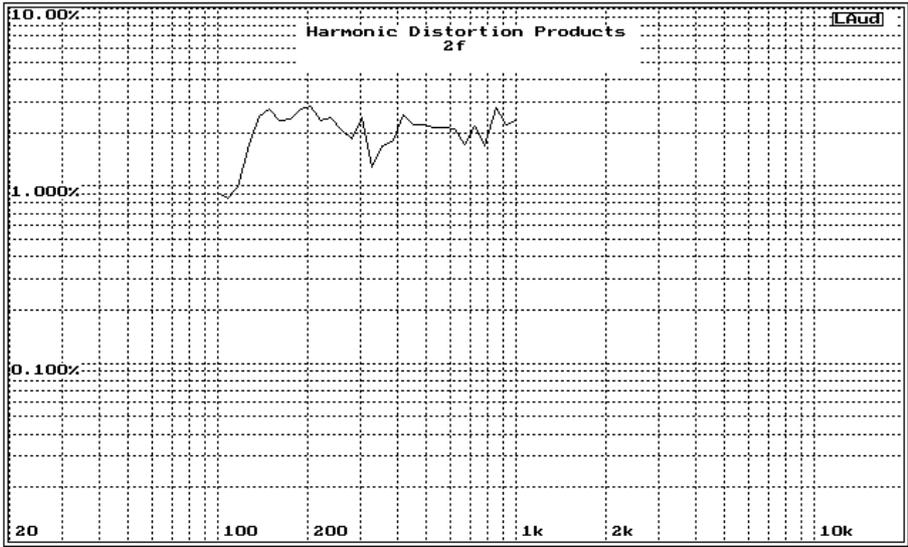


fig. 1: speaker 2nd harmonic distortion with transistor amplifier. This amplifier has less than 0.05% 2nd harmonic distortion when driving this load

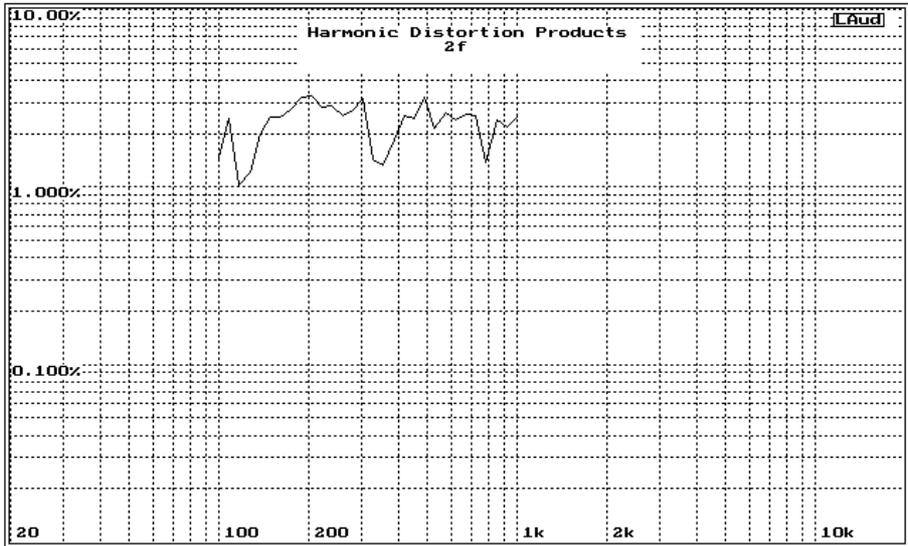


fig. 2: speaker 2nd harmonic distortion with SE amplifier. The SE amplifier has around 0.8% 2nd harmonic distortion when driving this load.

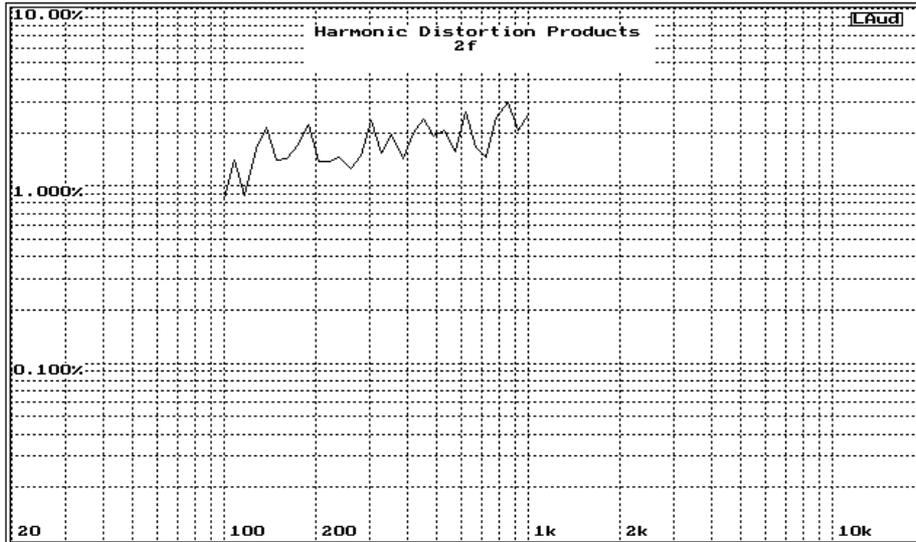


fig. 3: speaker 2nd harmonic distortion with the same SE amplifier. The SE amplifier has around 0.8% 2nd harmonic distortion when driving this load. The polarity is inverted compared to figure 2.

5. Some Consequences

Let's see some of the strange facts that we can explain based on what we have seen:

- **The preference for full range drivers to use with single ended amplifiers** is very easily explained. In a two way loudspeaker the crossover will direct the fundamental to one driver and the 2nd harmonic to the other over a range of frequencies. The drivers will not benefit from the effect I described over a fairly large range of frequencies. The crossover will also disrupt the original phase relation between harmonics and fundamental in the amplifier signal reaching the driver. This may be good or bad but it seems that it can do more harm than good. The dispersion characteristics and spatial positioning make very hard for the same effect to happen effectively in the acoustic side. Also the different amount of distortion of two different driver units increases the chance that at least one of them will not benefit from the effect. Loudspeakers with three or more crossover points can reduce this "single ended" effect considerably. This is also one of the advantages of using active crossovers.
- **Each loudspeaker, depending on its efficiency and distortion levels will mate better with single ended amplifiers of corresponding different power ratings and distortion** characteristics. The famous case of 2A3 amps sounding better than 300Bs with Lowthers could be such case.
- **Even technically correct global negative feedback when applied in small quantities can make the sound worse**, if the good sound of the system relies on this effect. The best amplifier for each loudspeaker is not a "perfect one" but one which has distortions that complements the speaker distortion. When negative feedback is introduced the single ended amplifier is no longer a simple electronic device and the increase in 2nd harmonic distortion with power will not follow so

- closely the increase in the loudspeaker distortion.
- **SE Amplifiers with less stages usually sound better.** The distortion interaction between stages can cancel the 2nd harmonic inside the amplifier and also make the distortion increase with power follow a more complex rule.

6. Some Problems

The 2nd Harmonic distortion is less offensive to the ears because of its musical relation to the fundamental but also because of the way it adds as it goes along a system. Single ended amplifiers can be made such that they will form a system with the loudspeaker that will explore this synergy fully. Obviously the injection of 2nd harmonic in any place of the chain can produce the same results if it can follow the behavior of the speaker distortion with frequency and power. This becomes progressively harder to achieve as we go backwards in a system. Single ended amplifiers are almost a natural way of doing it. Tube preamps or tube gadgets to "smooth" the sound of CDs can try, just as, may be, LPs do, but going further back in the chain is certainly wrong. Injecting second harmonic distortion in the recording is something that may help the sound of the recording in the particular system used for monitoring but it introduces forever something strange that certainly will not work with other systems.

7. The High Output Impedance

Another criticism which is often raised against single ended amplifiers is that they have a high output impedance. This is also a very important point because if an amplifier has a high output impedance the frequency response of the amplifier and loudspeaker together will be affected by the loudspeaker input impedance. This input impedance is normally anything but constant. I have described in great detail all that happens in two articles in Glass Audio (SE Amplifier Output Impedance I & II - GA 3/97 & GA 6/97). In these articles I have also considered the effects of the finite primary inductance and I have shown that in certain conditions this might help reduce the problems of the high output impedance changing the bass alignment of the loudspeaker. I believe that the two most important conclusions are:

- If the speaker has its low frequency design done with the high output impedance of a SE amplifier taken in account you can not only correct the small signal low end frequency response but actually get an added bonus with an improved relation between size, low frequency extension and efficiency.
- The values of the primary inductance of the output transformers usually can be smaller than calculated for a fixed resistor values if you are driving real world loudspeakers.
- The high output impedance can have several ill effects on a system, but if the system is designed taking it in account it will actually benefit from it. And certainly there are the cases where it just happen to "fix" a bad design.

8. Conclusion

The really good sonic performance of a well put together system using single ended amplifiers can be understood if we consider the whole system performance. A system made using single ended amplifiers is a perfectly valid way of achieving a very good sound also from the measurements point of view. We just have to measure the correct parameter. **The output of the whole system.** The other way of achieving a low distortion system is to use very low distortion loudspeakers with almost perfect amplifiers, everything used below

their full power range, but this tends to be also costly and may be at least as difficult to achieve as a correctly assembled SE system. If we take in account the distortion reducing mechanism and the low frequency advantage that the high output impedance may give, it is possible to design SE systems which can have a measured acoustic performance equivalent or better than the best examples of the traditional high end systems.

Appendix - How Do Distortions Add and Multiply

When we talk about harmonic distortion we usually say a number, like 1%. We need to remember that this number is an extreme simplification of what is going on and as we will see it may not represent anything at all without further information about it. First we need to know what harmonics we are talking about. 2nd, 3rd, 4th, what is the proportion of each of these. We need to know the spectrum of the distortion. We should have a number associated with each of the harmonics and if we look at the definition of total harmonic distortion we can see how we will add all the harmonics to arrive at one number that represents the THD.

$$\text{THD} = (H_2^2 + H_3^2 + H_4^2 + \dots) ^{1/2} \quad (1)$$

where H_2 = % of 2nd harmonic, H_3 = % of 3rd harmonic ...

If you do some calculations you will see that, just as an example, if we add 1% of 2nd Harmonic plus 1% of 3rd harmonic we will have a THD of 1.4%. But we may arrive at this 1.4% of THD in many ways. It may represent the summation of 1% of 2nd with 1% of 3rd as we said above or the same addition of any two harmonics. It also may simply be 1.4% of 2nd harmonic or 1.4% of 3rd harmonic or 1.4% of 10th harmonic. It may also represent that we have around 0.44% of each of ten harmonics (something like 2nd + 3rd + 4th ++ 11th)!! You can be sure that the same 1.4% would sound completely different in each case.

As you could see to add the harmonics to get the final THD figure we needed only the number which represents the quantity of each of the harmonics. But it is very easy to see why this THD number can not have any correlation with what we hear, except, perhaps, when comparing different THD numbers from devices with the same relation between harmonics.

But what happens if we need to add (actually multiply) the distortion from two sources. How can we calculate the final distortion of a system composed of an amplifier and a loudspeaker as in our case. Now things are a bit more strange. It is not enough to know the spectrum of the distortion of each of the two devices. We can not just add the 2nd harmonic of the first device with the 2nd harmonic of the second and so on, and arrive at a new spectrum. Although it is not necessary to calculate the total THD number from the spectrum we need one more information about each harmonic in the spectrum to actually multiply the distortion of two devices. We need to know the phase of the distortion of each harmonic relative to the fundamental in that particular device. Because we need to multiply each of the harmonics of the first device with all of the harmonics of the second devices taking the phase of each one in account. It requires only basic trigonometry and looking at the simplest case of all, multiplying the distortion of two devices which have only second harmonic distortion, is very useful. After all, loudspeakers and single ended amplifiers, as we already said, are devices which generate mostly 2nd harmonic distortion, quite a bit less of third and nothing else when used at power levels within their ratings.

Lets see what will happen when we connect two devices. First we will look at a distortionless amplifier followed by a loudspeaker which produces 2nd harmonic distortion. Then we will move on to the case of an amplifier also producing 2nd harmonic distortion followed by the same loudspeaker.

In the first case device #1 (the amplifier) is perfect.

For an input $x = E \cos \omega t$

(where $\omega = 2\pi f$, f = frequency and E = amplitude) (2)

The output will be:

$$y = Ax \rightarrow y = AE \cos \omega t, \text{ where } A \text{ is the gain. (3)}$$

This would be the case of the perfect wire with gain. Now we take the output and use it as input to the second device (loudspeaker) which will have 2nd harmonic distortion. The output w of the whole system will be:

$$w = By + ce^{i\Theta} y^2, \text{ where } B \text{ is the gain (4)}$$

c is a factor related to the amount of the 2nd harmonic distortion and the e to the power of $i\Theta$ represents the phase Θ which is assumed constant for each w . Substituting (3) in (4) :

$$w = BAE \cos \omega t + ce^{i\Theta} A^2 E^2 \cos^2 \omega t \quad (5)$$

Using trigonometry identities we will get the result:

$$w = BAE \cos \omega t + A^2 E^2 \frac{ce^{i\Theta}}{2} + A^2 E^2 \frac{ce^{i\Theta}}{2} \cos 2\omega t \quad (6) \text{ (output of the system)}$$

The first term represents the original signal multiplied by BA , the second term (which is independent of frequency (ω)) represents a DC component and the third term is the second harmonic with an amplitude of $A^2 E^2 c/2$ and a phase Θ .

It should be remembered that the phase angle Θ may be a function of frequency but here we are looking at just one frequency therefore it is a constant. Also, just one real world remark, the DC component, in our particular case, could displace the voice coil of the loudspeaker of its normal position but we should not be concerned with this right now. Now we will look at the output of a system where the amplifier and the speaker both produce 2nd harmonic distortion. y is the output of the amplifier for an input like (2) :

$$y = Ax + de^{i\Phi} x^2 \rightarrow y = AE \cos \omega t + de^{i\Phi} E^2 \cos^2 \omega t \quad (7)$$

where d is the factor which defines the amount of the 2nd harmonic distortion and Φ is the phase of this second harmonic in the amplifier. Again using the same trigonometric identities we get:

$$y = AE \cos \omega t + E^2 \frac{de^{i\Phi}}{2} + E^2 \frac{de^{i\Phi}}{2} \cos 2\omega t \quad (8)$$

We will ignore the second term. It is DC and depending where the distortion is generated inside the amplifier and the frequency response after it, it will probably not be at the output. Therefore we consider the output of the amplifier to be:

$$y = AE\cos\omega t + \frac{E^2 d e^{i\Phi}}{2} \cos 2\omega t \quad (9) \text{ (output with 2nd harmonic)}$$

This is the amplifier output which we will connect to the loudspeaker: Substituting (9) in (4) will give us equation (10) :

$$w = BAE\cos\omega t + \frac{BE^2 d e^{i\Phi}}{2} \cos 2\omega t + c e^{i\Theta} (AE\cos\omega t + \frac{E^2 d e^{i\Phi}}{2} \cos 2\omega t)^2$$

Expanding the terms inside the parenthesis squared, using a little trigonometry and rearranging the terms we will get:

$$\begin{aligned} w = & (BAE + \frac{c d e^{i(\Theta+\Phi)}}{2} AE^3) \cos\omega t + \text{(fundamental)} \\ & + (\frac{BE^2 d e^{i\Phi}}{2} + A^2 E^2 \frac{c e^{i\Theta}}{2}) \cos 2\omega t + \text{(2nd harmonic)} \\ & + \frac{AE^3 c d e^{i(\Theta+\Phi)}}{2} \cos 3\omega t + \text{(3rd harmonic)} \\ & + \frac{E^4 c d^2 e^{i\Theta}}{8} \cos 4\omega t + \text{(4th harmonic)} \\ & + \frac{A^2 E^2 c e^{i\Theta}}{2} + \frac{E^4 c d^2 e^{i\Theta}}{8} \quad \text{(DC component)} \end{aligned}$$

As we can see the output even for this simple 2nd harmonic case is not simple. Lets look at the 2nd term:

- The second term is the 2nd harmonic distortion. It will be reduced to zero if :

$$(\frac{BE^2 d e^{i\Phi}}{2} + A^2 E^2 \frac{c e^{i\Theta}}{2}) = 0$$

For this to happen we need that $B d = A^2 c$ and that $\Phi - \Theta = 180^\circ$.

This actually means that we need an amplifier that produces the same amount of 2nd harmonic distortion as the loudspeaker at the same power levels and that the difference between the relative phases of these distortions is 180° . But, more important, it also means that if the amplifier and the loudspeaker have comparable 2nd harmonic distortion specifications and differences in the relative phases bigger than 120° we will have a distortion reduction that can be very significant.

Here I need to make comments about this derivation and the values of the other terms:

- The assumption that the distortion producing mechanism is not DC coupled to the output is an important and very reasonable one at mid frequencies for the case of

SE amplifiers with output transformers. Anyway if the distortion producing mechanisms are DC coupled we can not have the ideal case of total 2nd harmonic distortion cancellation because a term with an E^4 in the coefficient will appear. Anyway this should be a very small value except at very high signal levels.

- The terms which correspond to the 3rd and 4th harmonics generated by the multiplication also have a much lower value. Not only they are multiplied by cd or cd^2 but they depend on E^3 or E^4 . Therefore at lower signal levels, were most of the music is, they are really very small compared to the 2nd harmonic term.